# Examiners' Report
# Principal Examiner Feedback

January 2021

Pearson Edexcel International A Level
In Pure Mathematics 2 (WMA12)
Paper : WMA12 / 01

## General

The paper provided plenty of scope for candidates to demonstrate their ability with some challenges to test the higher grades. The question on proof was found the most difficult on the paper, with candidates still becoming familiar with the newness of the topic on this specification.

Questions 1, 2, 4,6 and 7 all had standard procedures that candidates are very familiar with to enable them to get started. Questions 5, 8, 9 and 10 each had discriminating parts to test at higher grades, though with question 10 the main challenge for candidates was unravelling the context.

There were a significant number of blank or near blank responses on the paper, probably due at least in part to the current pandemic affecting many areas of life. About 5%, perhaps, were scripts with little or no progress, with the highest scoring trait on the paper having a 93.5% success rate. Statistics quoted in this report need to have this borne in mind.

## Report to individual questions

## Question 1

In general, this question was well done and most candidates understood what was required with nearly 75% scoring full marks on this question and roughly 90% recognising this as a remainder theorem question and accessing the first marks in each part.

In part (a) the substitution and rearrangement of $f(-1) = 4$ were usually shown clearly with very few losing the accuracy mark due to sign errors or bracketing errors. There were a few sign slips but approximately 85% of candidates were able to achieve both marks here. Only a small minority tried to use the overly complicated approach of long division, rather than the Remainder Theorem which overcomplicated the process and often led to errors.

Again in part (b), the majority of candidates were able to substitute 2 in to $f(x)$, use $f(2) = -23$ and then solve the simultaneous equations. A few equated to 23 rather than −23, and there were a few sign errors, errors in rearranging and some copying errors. Method marks were almost always picked up in even when accuracy was lost. Errors included simplifying the second equation correctly and some errors in solving the pair of equations. Using a calculator to solve was rare but acceptable. Again the few who attempted long division were usually unable to complete the question successfully.

The alternative method $f(x) = (x + 1)$ (cubic) + 4 was seen only occasionally, with traditional use of the remainder theorem with predominant approach.

**Question 2**

Part (a) was generally very well answered, though there were a number of non-response scripts, with only about 90% offering an answer. Of these the vast majority when on to score full marks in this part. Most candidates knew to differentiate the given function, set the derivative equal to zero and find the *x* coordinates of the stationary points. There were occasional slips in coefficients of terms or signs, but this was in a very limited number of cases. However, candidates often went on to do unnecessary work by finding the *y* coordinates of the stationary points or assumed this was what was required of part (b).

In part (b), although most (roughly three quarters) candidates recognised that they had to find the second derivative there were many who did not then substitute the values for *x* found in part (a) into their second derivative. A common error was to try to solve for the second derivative equal to zero, while others correctly substituted $\frac{8}{3}$ and $-2$ to achieve 14 and $-14$ but gave unsuitable interpretations such as “increasing” and “decreasing”. Fewer than two thirds manage the second method mark for substituting and drawing a conclusion. Candidates often missed to complete their conclusion by indicating that it was the sign of their second derivative that was used to determine the nature of the stationary point and only 50% scored the final A mark.

Only a very small minority of candidates chose to examine the sign of the gradient on either side of the stationary point in order to determine its nature and this method was given full credit as it used further calculus. A rarely used method looking at the shape of the curve by looking at the *y* values either side of the stationary point was not given credit as it did not use further calculus as asked for in the question.

## Question 3

While many candidates demonstrated some knowledge of logarithms, only the strongest candidates were able to achieve full marks in both parts of this question, with the final mark in (ii) being particularly discriminating.

Most candidates were able to make some progress in part (i). The most common approach was to take logs on both sides (usually base 7) to reach $x + 2 = \log_7 3$. The mark for making $x$ the subject in an exact form should have been straightforward but some chose to introduce decimals while others were unable to correct rearrange. A fall off from nearly 90% to just over 70% between the first two marks is revealing, with only 50% scoring all three. For example, may achieved $x = \log_7 3 - 2$ but then did not reach the required form, failing to deal with −2. Those who chose the indices route mainly produced clear concise work, particularly if they replaced $7^2$ by 49 before rearranging. Although the question asked for an answer in log form, it is not always advisable to take logs first when the equation is presented in index form.

Part (ii) was a more discriminating question with fewer than a third scoring full marks. About 80% could correctly employ at least one logarithm law (usually by applying the addition rule to any two terms but also for a very small minority it was for using $\log_2 2 = 1$ in some form) to make progress with this question but only 70% managed to correctly remove logarithms from the equation. Some of these made an error in applying one or more of the rules and were unable to achieve a correct quadratic equation, while others simply did not know how to proceed and stopped. Common incorrect working included, for example, $\log(y + 4) = \log y + \log 4$ and similar incorrect splitting of logs. It was common to see candidates simply removing all logs to get $1 + y + y + 4 = 5 - y$.
Of those who managed to achieve a quadratic from acceptable work, nearly all correctly solved their equation (often by calculator). However, only a minority realised that the presence of $\log(y)$ in the original question forced $y > 0$ and so many otherwise correct solutions lost the final answer mark as they failed to reject −5 as an answer. A few wrote $y = \frac{1}{2}$ following incorrect working or virtually no working suggesting they had used the equation solver or graphical functions on their calculator.

**Question 4**

Binomial expansion is a well understood topic with nearly all students making some progress in part (a). The application in part (b) was discriminating in the mid grades, with the ideas of identifying relevant coefficients unclear to many.

In part (a), most candidates expanded using the formula for $(a + b)^n$ with very few instances seen of the factor $2^6$ being taken out first. A small minority attempt expansion of brackets piecemeal, usually resulting in errors, as well as finding many more terms than needed.

Over 90% of candidates manage one or both of the first two marks, with the B mark for achieving the 64 being most successfully achieved. The most common error seen was in the third term which was often given a $p\,x^2$ rather than $p^2x^2$. The alternative of $(px)^2$ was allowed for full credit.

Part (b) was far more demanding, with little over two thirds making progress, though 50% in total achieved the correct answer. Most realised the need to try and expand the two brackets, with only the most able students able to identify the correct two coefficients without a full expansion. The expansion was generally well done and the two correct $x^2$ terms identified, though some candidates only focussed on one term in $x^2$ and hence score no marks. A small number ended up with more than two terms in $x^2$ instead.

Common errors after the attempt at expanding (whether successful or not) were: to equate each term to $-\frac{3}{4}$ and solve two equations, rather than equating the sum of the coefficients of the terms to get a quadratic in $p$; leaving the $x^2$ term in the quadratic then substituting $-\frac{3}{4}$ for $x$; equating the sum of the two $x^2$ terms to $-\frac{3}{4}$ rather than equating the sum of the coefficients to $-\frac{3}{4}$.

Even when candidates managed to identify the correct two terms and equate the coefficient of $x^2$ to $-\frac{3}{4}$, not all ended up with a quadratic in $p$, as the common mistake highlight in part (a) of $px^2$ meant they only ended up with a linear term. Candidates who did achieve a quadratic in $p$ from suitable work almost always scored the method for solving the quadratic, and if the correct equation had been achieved scored full marks.

## Question 5

This was the most challenging, and least well answered, question on the paper, with part (i) particularly discriminating at the high grades with only 45% of candidates getting started, less than a third achieving the second method and only a tenth completing the proof. Even the production of a counter example for (ii) was only successfully achieved by 40%.

For part (i) some candidates could produce the necessary evidence required for a proof, but many of these did not then complete the proof with a conclusion.

The most efficient solutions moved from a starting point $(\sqrt{(3x)} - 1)^2 \geq 0$ to the required inequality; this was seen rarely, however. Some were able to work "backwards" from the given inequality to get to $(\sqrt{(3x)} - 1)^2 \geq 0$ and then worked "forward" to the required inequality.

Most candidates attempted the alternative method of attempting to square and rearrange the required inequality, however even here many did not achieve three terms in the square on the left side of the inequality. Some did reach a perfect square $(3x - 1)^2 \geq 0$, but rarely achieved a full proof as it then required some reasoning. They either stopped at this inequality or continued with $x = \frac{1}{3}$ so $x \geq 0$, and not referring back to the original inequality. Those that considered the determinant of the resultant quadratic rarely scored the second mark due to a failure to consider the "positive nature" of the quadratic expression.

Others rearranged without squaring first but recognised it as a quadratic in $\sqrt{3x}$ which could also gain the first mark (a variant on Alt 2). Again here, though some were able to factorise to a perfect square, or in some case following error, achieve a non-negative expression via completion of the square, a formal conclusion was often lacking. Setting up a contradiction proof formally was seldom seen.

A common erroneous answer was to try and demonstrate that the result was true by testing some values of $x$. There were also many scripts either blank or with no valid method scoring 0 marks. Another common error was that some candidates failed to appreciate that the $x$ on the right hand side was included under the square root sign, though many such candidates were able to at least pick up the method marks.

For part (ii), the majority of candidates were aware that a proof by counter example was required, but a significant minority of them were unable to find three consecutive primes.

Those who understood the question and knew what the words meant completed this easily.

However, the number of candidates who did not know what "sum", "consecutive" and/or "prime" meant was rather surprising. There seems a real confusion between the meaning of prime and odd number, a few even used three even numbers. A few found the product of three primes. Another common error was to use three primes which were non-consecutive. The use of 1 as a prime was another common error, as also was use of 9. A few tried to prove using algebra, using eg $2n+1$, $2n+3$, $2n+5$.

There were some candidates who seemed to think that this part followed on from part (i) and so tried using primes in the inequality of part (i).

Once again, a lack of rigour in proof meant that some candidates failed to conclude, even minimally, despite producing a correct example.

**Question 6**

Although there were some candidates who made little or no attempt at this question with about 15% failing to get started, overall, it was reasonably well done aside from accuracy issues.

Part (a) was a standard and expected trigonometric proof question. Most candidates attempted the procedure to use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ in the given equation, multiply both sides by their denominators and use the identity $\cos^2\theta = 1 - \sin^2\theta$ to form an equation in $\sin\theta$ only and 60% were able to proceed to the given answer correctly and achieved full marks.

The common errors where things went wrong were: stating $5\tan\theta = \dfrac{5\sin\theta}{5\cos\theta}$ with no correct identity for tan $\theta$ given; errors when multiplying through by the denominators, such as $(3\sin\theta\cos\theta)\cos\theta = 3\sin\theta\cos\theta\cos^2\theta$; attempting to substitute for $\cos\theta$ before multiplying by the denominators usually using incorrectly $\cos\theta = 1 - \sin\theta$; incorrect substitution of $\cos^2\theta = (1-\sin\theta)^2$;incorrect bracketing such as $1 - \sin\theta(5\sin\theta)$; incorrect consistent poor notation $\sin\theta^2$ or sin lead to withholding the final accuracy mark.

Part (b) was less well answered than part (a) with only 75% making a suitable start and less than 50% success rate on each of the final two marks (often with candidates gaining one or the other but not both).

Candidates generally read the question carefully and noted Hence and used the answer to part (a). Most attempted to solve the cubic and achieved a root of $\dfrac{2}{3}$, usually gaining the first two marks (with exception being those who gave $x = \dfrac{2}{3}$ as the final answer). Many then proceeded to $\sin^{-1}\left(\dfrac{2}{3}\right) = 0.7297$, however, several candidates gave this as their answer and did not divide by 2 to find the value of $x$. Occasionally they multiplied by 2. A few gave an additional solution in the range, usually $x = -0.365$ and a few had found answers in degrees and not changed them into radians. Candidates are advised to work in radians and check the requirement of the question.

The most common mistake was to miss out the answer $x = 0$. Some because they had cancelled their cubic earlier. Others had factorised earlier and had $\sin 2x = 0$ or $\sin\theta = 0$ but either forgot to proceed and identify $x = 0$ or left as $\theta = 0$ and not $x$.

**Question 7**

This was familiar topic in a context, but the context of the question did not affect candidates, and it was usually well answered.

The first B mark in part (a) was the most successfully achieved mark on the entire paper at 93.5% success, with incorrect values being very rare; blank responses accounted for most of the failures to score this mark. A few more lost the second mark, though, either by a completely incorrect value, or by giving the value to only 2 d.p., although these latter candidates often went on to gain full marks in (b).

Part (b) was successfully completed by about 60% of candidates. The structure of the Trapezium Rule was understood by most with about 80% scoring the M1A1ft marks. The main reasons for losing the method mark were the usual ones: $t$ values instead of $P$ values used in the brackets; repeated values (usually including the "last value" too many time); missing outer brackets that were no recovered. The latter of these could be recovered but there were cases where only the "first +last" terms were multiplied by $h$.

The most common error overall continues to be using/identifying an incorrect value for "$h$", though many gained the marks for the structure following this error. Candidates often confuse the number of strips with the number of ordinates and use $h = \frac{12-10}{5}$ in error, without realising that the width $h$ is simply the difference between the measured $t$ values, 0.5, and can be read directly from the table.

The final mark required the specified accuracy, and a few lost the accuracy mark by not giving the answer to 2dp.

**Question 8**

This proved a question with a mixed response from candidates and proved the second hardest question after question 5. Some candidates were clearly poorly practiced for sequence questions of this type and attempted to fit Arithmetic and/or Geometric sequences to the given sequence.

Part (a) was successfully completed by over 80% of candidates, though many to several lines to find the correct expression for $a_2$. Attempts at expanding $2\left((p-3)^2+6(p-3)+9\right)-7$ were seen often, usually resulting in error or incomplete simplification, where the intent was for candidates to spot the plus and minus 3's cancel directly. A few did not recognise or understand recurrence relations at all and made no progress.

Part (b) was more problematic and was omitted by quite a few candidates who had successfully answered part (a). About two thirds made significant progress, but only a quarter managed full marks.

Candidates who achieved the mark in (a) often went on to correctly find $a_3$ (if they progressed at all), although there were some who made algebraic slips in simplifying $a_3$, especially with with squaring $2p^2-4$

About 15% of candidates who worked out $a_2$ did not then know how to proceed correctly, again possibly a lack of practice on questions involving the summation notation for recurrence relations, with only 40% in all attempting the sum and setting to $p + 15$ correctly. A slip using $p+5$ was seen fairly often but was permitted the method. On a few occasions the first three terms were added successfully but then a failure to put the sum equal to $p+15$ meant no further progress could be made, with some instead equating $p + 15$ to $a_3$, or even $a_2$ or just $p - 3$ (or all three independently).

Those candidates who successfully obtained the quartic of $8p^4-30p^2=0$ usually went on to solve it successfully though the solution $p^2=0$ was sometimes lost, for example by dividing through by $p^2$ instead of factorising or using their calculator to find the values for $p$. Candidates need to pay special attention that solutions where a variable is equal to zero need to be considered carefully.

Some candidates who successfully found at least one value for $p$ failed to again the final method mark for the question by not following through to find both values for $a_2$, instead often thinking the values of $p$ were the answers, so not being aware of what the question was asking. Only a small minority used an incorrect formula for $a_2$. A few, however, only found one value for $a_2$ sometime even when both solutions for $p^2$ had been found.

**Question 9**

The first two parts of this questions were familiar to students and provided a good source of marks to all, with 80% successfully answer part (a) and around 85% in part (b). Part (c) proved a very good discriminator, with only around 50% of candidates making significant progress, and fewer than 25% achieving a correct final answer.

In part (a) the majority candidates achieved two marks for correctly writing down the centre coordinate and the radius. The few errors that were common were for the centre $(-k.-2k)$, $\left(\frac{k}{2},k\right)$ being given, and for the radius, not taking the square root of $k+7$ or even $(k+7)^2$. A few candidates left their answer as $r^2=k+7$.

Part (b) was another routine part giving access candidates of all abilities. Most had no problem substituting in and expanding to achieve desired result. Sign errors and achieving only 2 terms when squaring brackets were the main reasons for losing the A mark. Finding the values for *k* was also well done by most candidates. A variety of methods of factorising, using quadratic formula and using a calculator were seen. There were some candidates who thought that the answer was a range of values of *k* rather than two distinct values, and so lost the B mark as such subsequent working was not ignored in these cases.

The answers in part (c) were more varied, with this proving a good discriminator question. Candidates largely attempted part (c) without drawing a sketch of the situation. A sketch is very helpful to the candidate in seeing what is required to solve the problem and it is advisable for candidates to use a sketch to help solve problems of this type.
After nearly 90% of candidates successfully navigating the first two parts, fewer than 60% managed even the first mark, for identifying the centre of the circle (either directly or using it to attempt the gradient), and only 50% made further progress. A few candidates chose the wrong value of *k* and so lost the mark, while others attempted to use the original equation to find where the circle meets the *x* axis thus making no progress at all with the question. Others simply putt the value of *k* into the equation of the circle and then stopped. This did not gain any marks.

Of the solutions that made progress most began by finding the gradient of the radius and using that to find the gradient of the tangent and then its equation, though a small number were able to find the gradient of the tangent directly. Very few attempted to differentiate the equation of the circle, and these often made little progress even though the next method marks were available.

The majority who attempted the tangent gradient then followed an attempt at finding the intercept on the $x$-axis, though a few found the $y$ intercept instead. When finding the equation of the tangent both $y = mx + c$ and $y - y_1 = m(x - x_1)$ were seen with equal frequency before substituting in $y = 0$ to find an $x$ value for the point T.

The final step was to find the area of the required triangle and was only achieved by the better candidates, roughly a third achieving the method. This is a place where a good sketch would benefit candidates. It was surprising to see many candidates not using the simple formula for the area of the triangle electing instead to use ½$ab$sin$C$. Another variant used in a minority of cases was the shoelace formula. These result from candidates not appreciating the geometry of the situation, with the requisite triangle being right-angled. Conversely, many candidates incorrectly assumed a right angle where there wasn't one and therefore did not achieve this mark, using $\frac{1}{2}OP \times PT$. These candidates assumed that $OP$ went through the centre and so failed to calculate the correct area.

Also seen frequently was the misunderstanding that $O$ was the centre of the circle rather than the origin. It is unfortunate that the question did not make this clear, though the use of $O$ as the origin is very standard notation that candidates are expected to know. In such cases a special case was permitted, and candidates were able to score all but the final A mark as work in evaluating their incorrect triangle was largely the same as for the correct triangle.

**Question 10**

This question was very well answered by many candidates though very few managed to score full marks. This question presented the candidate with a large amount of information to be absorbed. The lengthy text had to be processed. Unfortunately, this led to confusion for some candidates. This was particularly true in part (b) where a significant number failed to recognise that they were dealing with a geometric series.

There was some evidence that some candidates ran out of time, but most were able to attempt the whole question, though little over 50% of candidates score marks in part (c), compared to over three quarters making a start in part (a).

It was generally recognised that part (a) was about an arithmetic progression and the correct answer was achieved by nearly 70% of candidates. Some divided (37 − 15) by 12 instead of 11 when finding the common difference but were able to score both method marks from this.

Part (b) was less successfully completed, but most (approximately 60%) recognised this as a geometric progression though less than half of candidates went on to get the correct answer. Some used $r^{12} = 37/15$ instead of $r^{11}$ and lost the first mark. Indeed, the second M was more successfully achieved than the first in both (a) and (b). It was very common to see $r =$ 1.0855... rounded to 1.09 in the calculation for $u_5$ leading to an accuracy error in the answer. Some correctly interpreted $r$ as an 8.6% increase but were then unable to successfully find $u_5$.

In part (c) about half of the candidates scored one or both of the first two marks for using the correct formulae for the sum of the first 12 terms for models A and B. Of those not scoring it was mostly due to omission, though a few cases of incorrect formulae being used were see too, for instance in thinking that it was the value of $n$ or $a$ that they were finding and so setting up an equation with these as a variable rather than using the correct values for them. For finding the sum of the arithmetic series, most used the standard formula with $a$, $n$ and $d$, but there was a significant minority who spotted that $a$ and $l$ were given in the question so the simpler equation $\frac{n}{2}(a+l)$ could be used.

Many of those who calculated the sums correctly went on to add $12x$ and equate to 360 or carry out equivalent work. However, it was common to see candidates adding $x$ rather than $12x$, thus getting an answer of 48 for model A and 64 for model B, accounting for most of the 10% drop in students who successfully found a sum but failed to score the third M.

Some interpreted "will not exceed 360" to mean less than 360 rather than less than or equal to 360. This caused some candidates to reject the correct answer of $x = 4$ and give $x = 3$ instead, though in many cases they lost the final A through error in Model B, so this did not affect their score. The condition that "$x$ is an integer" wasn't noticed by some which cost them the final accuracy mark with the answer 5.4... often given for Model B rather than $x = 5$.
For candidates using $r$=1.09 they often still gained the 1st A mark as they correctly found the 4 km required for model A, and so lost just the final A mark.
With a 15% success rate, the final A mark proved the second hardest mark on the paper, behind only the accuracy in question 5(i), though over a third were successful in achieving the penultimate mark.
Throughout the question it was good to see the vast majority of candidates using the term and summation formulas for series rather than listing terms.